

# Tests of Gravity and Dark Energy with Cosmology & Gravitational Waves

## Lecture 1: theoretical foundations and cosmological tests

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BERKELEY CENTER *for*  
COSMOLOGICAL PHYSICS



Mesoamerican Center for Theoretical Physics

October 2018

Further reading

- “Dark Energy in light of Multi-Messenger GW astronomy” 1807.09241
- “*hi-class: Horndeski in the Cosmic Linear Anisotropy Solving System*” 1605.06102
  - “Modified Gravity and Cosmology” 1402.5031

**gravity**

'gravɪtɪ/

*noun*

1. [Physics]  
the force that attracts a body towards the centre of the earth, or towards any other physical body having mass.
2. extreme importance; seriousness.

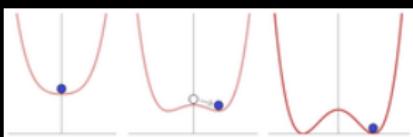
Sources: google (1,2)

# The case for beyond-GR theories

- Interesting theoretical questions:

$\sim 36\%$  of open problems in physics involve gravity

(see [www.wikipedia.org/wiki/List\\_of\\_unsolved\\_problems\\_in\\_physics](http://www.wikipedia.org/wiki/List_of_unsolved_problems_in_physics))

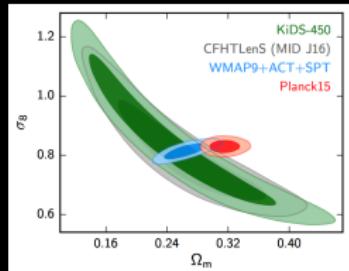


- cosmological constant problems?
- proxy for inflation/quantum gravity?

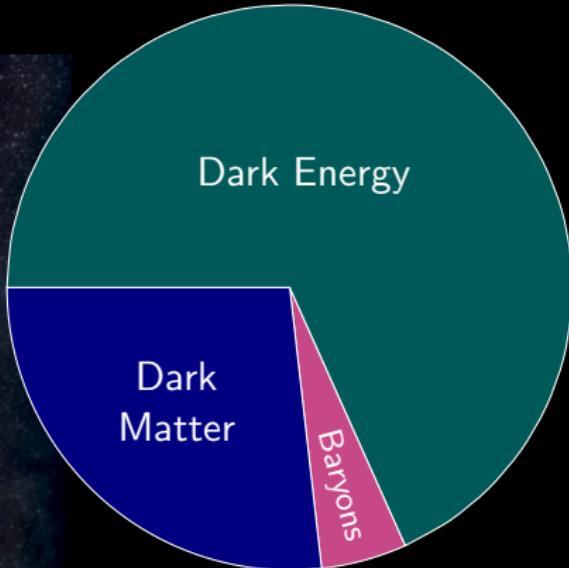
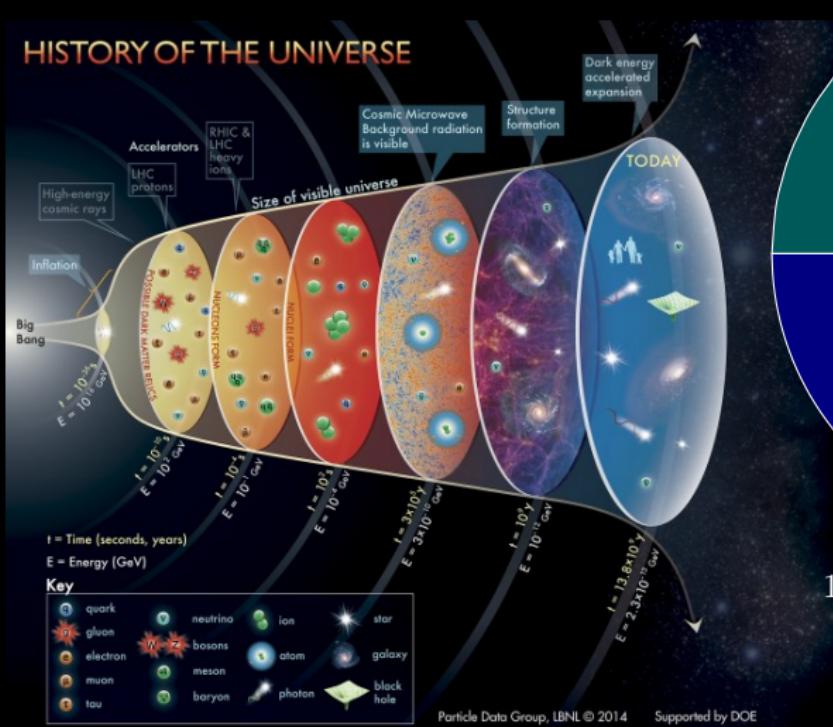
- Alternative models

- $\Lambda$ : Inflation again? (end,  $n_s \neq 1$ )
- $\Lambda$ CDM tensions  $\longrightarrow$   
(weak lensing, Hubble rate, clusters...)

- Test gravity on all regimes



# The Dark Universe



$$100\Omega_bh^2 = 2.222 \pm 0.023 \text{ (1.0\%)}$$

$$\Omega_c h^2 = 0.1197 \pm 0.0022 \text{ (1.8\%)}$$

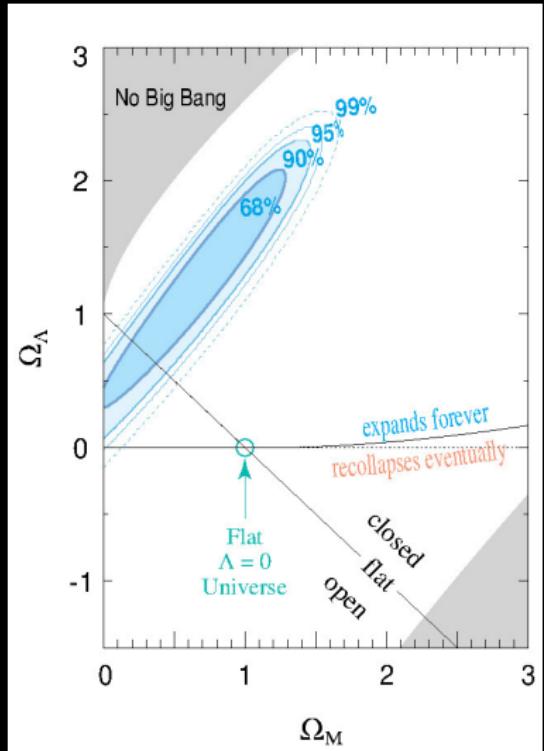
$$\Omega_\Lambda = 0.685 \pm 0.013 \text{ (1.9\%)}$$

Planck '15 (T+lowP only!)

Well understood laws and history...

...but important unknowns

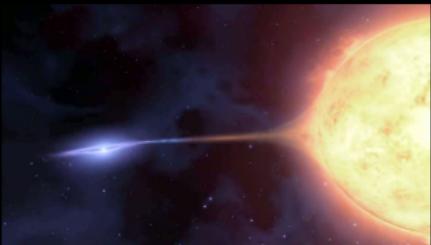
# Cosmic Concordance



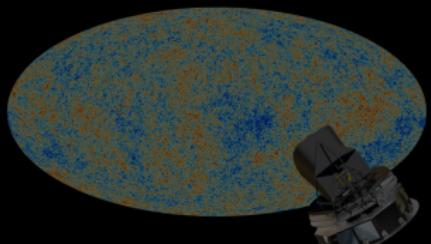
(Perlmutter+ 99, Amanullah+ 2010, Scolnic+ 2017)

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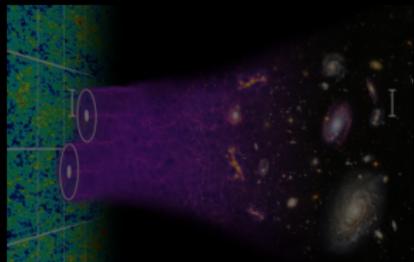
# Type IA Supernovae



# Cosmic Microwave Background

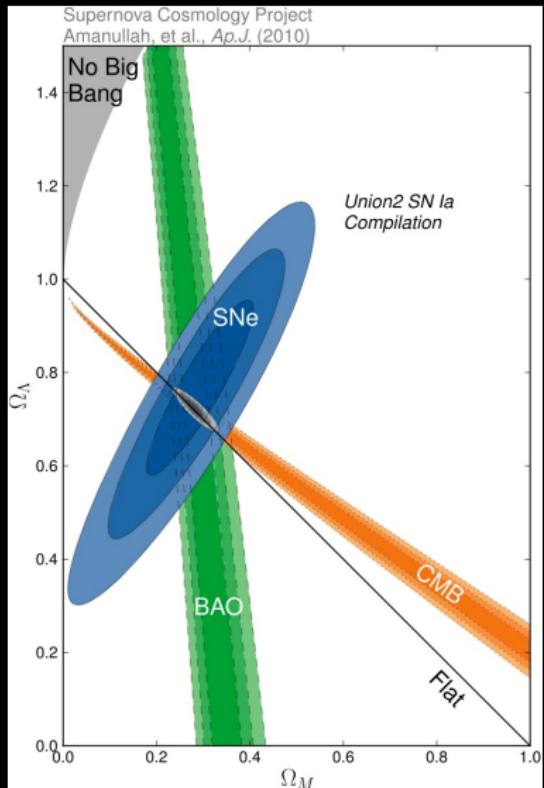


# Baryon Acoustic Oscillations



Tests of Gravity & DE with cosmology & GWs

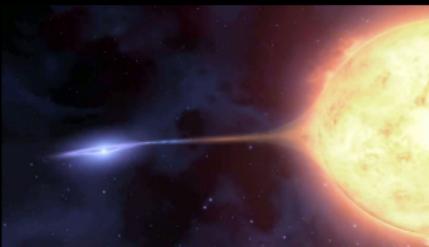
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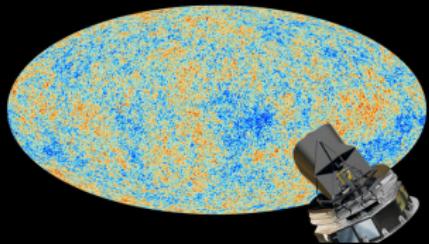
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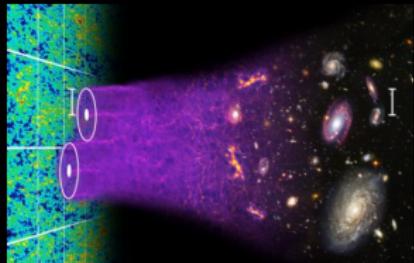
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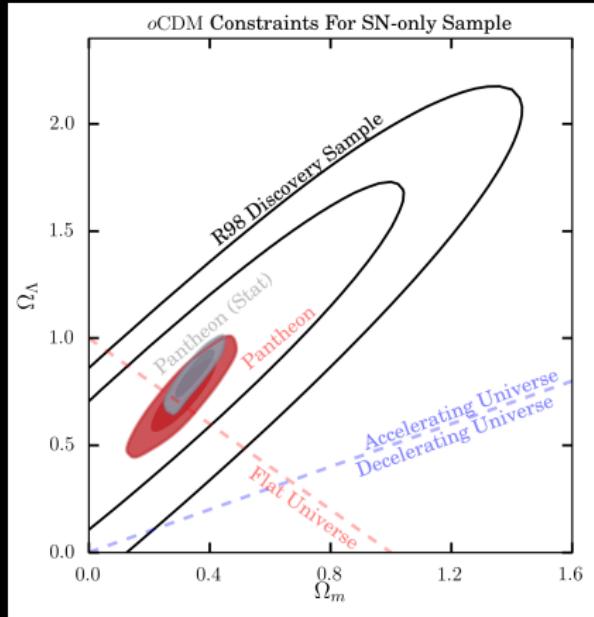
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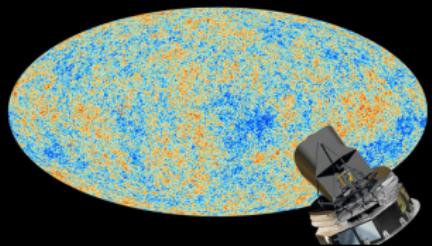
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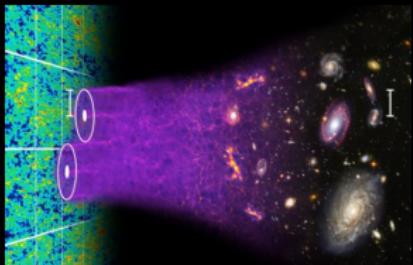


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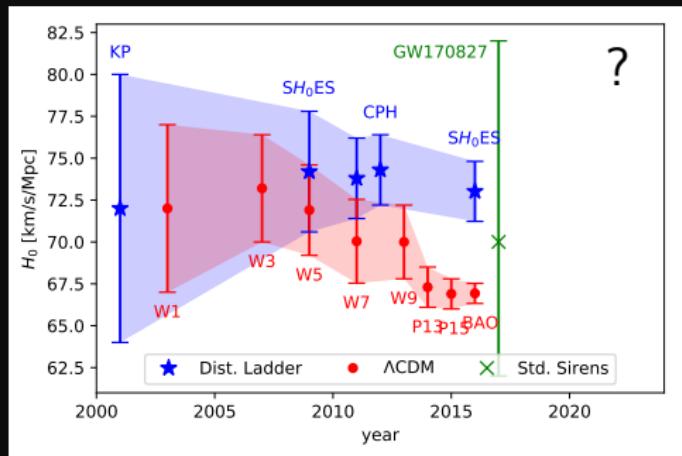


## Baryon Acoustic Oscillations



# $\Lambda$ CDM tensions $\rightarrow$ systematics or new physics?

Cosmic expansion:  $3.4\sigma$  tension in  $H_0$



(Adapted from Freedman '17, 1604.01788, 1710.05835)

- Distance ladder
  - + several reanalyses
  - + lensing time delays
- CMB (+BAO)  
→ assumes  $\Lambda$ CDM
- Standard Sirens
  - ✓ independent phys.
  - 15× BNS → 1–5%

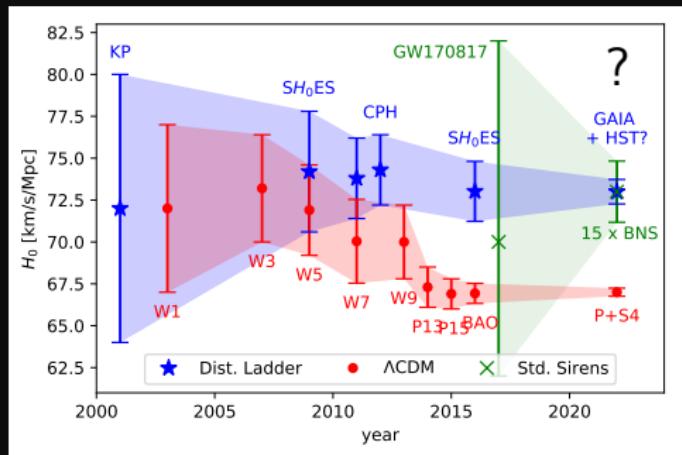
(Nissanke+ 1307.2638)

No simple explanation (Bernal+ '16, Poulin+ '18)

Also: Weak Gravitational Lensing (KiDS:  $2.9\sigma$ , DES: OK?)  
Clusters, Planck lensing...

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# The standard cosmological model

## Theory of Gravity

$$G_{\mu\nu}[g_{\mu\nu}] = 8\pi G T_{\mu\nu}$$

Einstein's GR

Cosmology requires new-physics!

Evidence for DM & DE relies on GR → independently test gravity!

# The standard cosmological model

## Metric Symmetries

$$-dt^2 + a(t)^2 d\vec{x}^2$$

Homogeneous & isotropic

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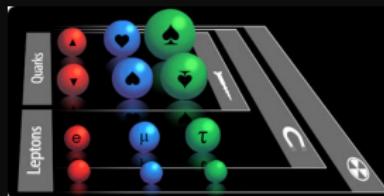
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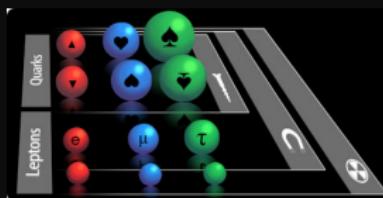
Modified gravity  
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## Structure formation

### Dark Matter

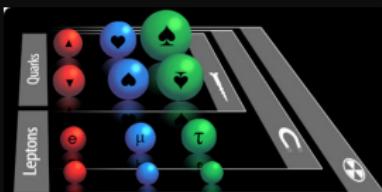
Modified Newtonian  
Dynamics?

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## Acceleration

$\Lambda$

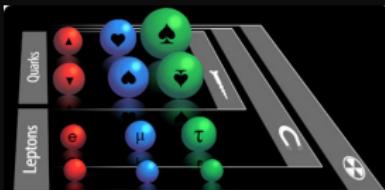
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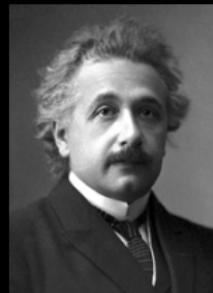
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# General relativity

$$S = \int d^4x \frac{1}{16\pi G} \sqrt{-g} R [\underbrace{g_{\mu\nu}}_{\text{metric}}]$$



Equations

$$\underbrace{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R}_{\text{curvature}} = 8\pi G \underbrace{T_{\mu\nu}}_{\text{matter}}$$

Cosmology:  $g_{\mu\nu} = \text{diag}(-1, a(t), a(t), a(t))$  (flat FRW)

$$H \equiv \frac{\dot{a}}{a} = \left( \frac{8\pi G}{3} \sum \rho_i \right)^{\frac{1}{2}} \rightarrow \rho_i \supset \text{matter+light, } \nu, \text{ DM, DE}$$

$$\ddot{a} = -\frac{4\pi G}{3} \sum (\rho_i + 3p_i) \rightarrow \text{acceleration} \Leftrightarrow p < \rho/3$$

# How to modify gravity

$$\underbrace{\sqrt{-g} \left\{ \frac{1}{16\pi G} R[g_{\mu\nu}] + \mathcal{L}_m \right\}}_{\text{Theory (Lagrangian)}} \rightarrow \underbrace{G_{\mu\nu} = 8\pi G T_{\mu\nu}}_{\text{Equations}} \rightarrow \underbrace{H(t), P(k, t)}_{\text{Solutions}}$$

GR → unique theory of massless spin-2, Lorentz, local, 4D...

QM+Lorentz (Weinberg '64), 2<sup>nd</sup> order eqs. (Lovelock '71)

Modify the recipe: (Clifton, Ferreira, Padilla, Skordis '11)

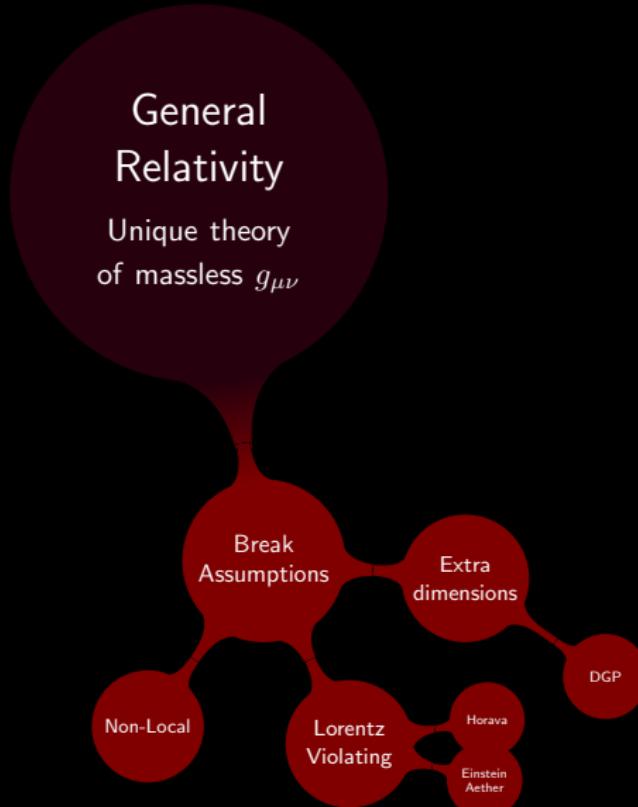
- New ingredients (degrees of freedom):
  - Massive gravity:  $2 \rightarrow 2s + 1 = 5$  polarizations  
→ (problems for cosmology)
  - Additional fields (scalar, vector, tensor...)
- New rules: Lorentz violation, Non-local, extra-dimensions, ...

# How to modify gravity

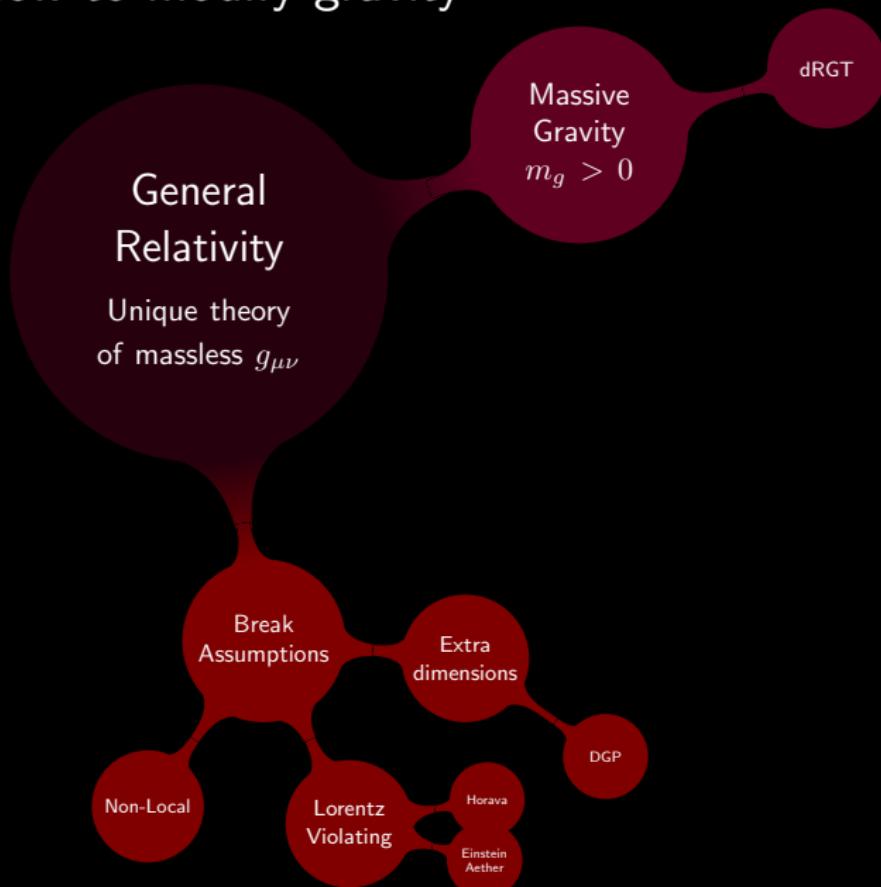
General  
Relativity

Unique theory  
of massless  $g_{\mu\nu}$

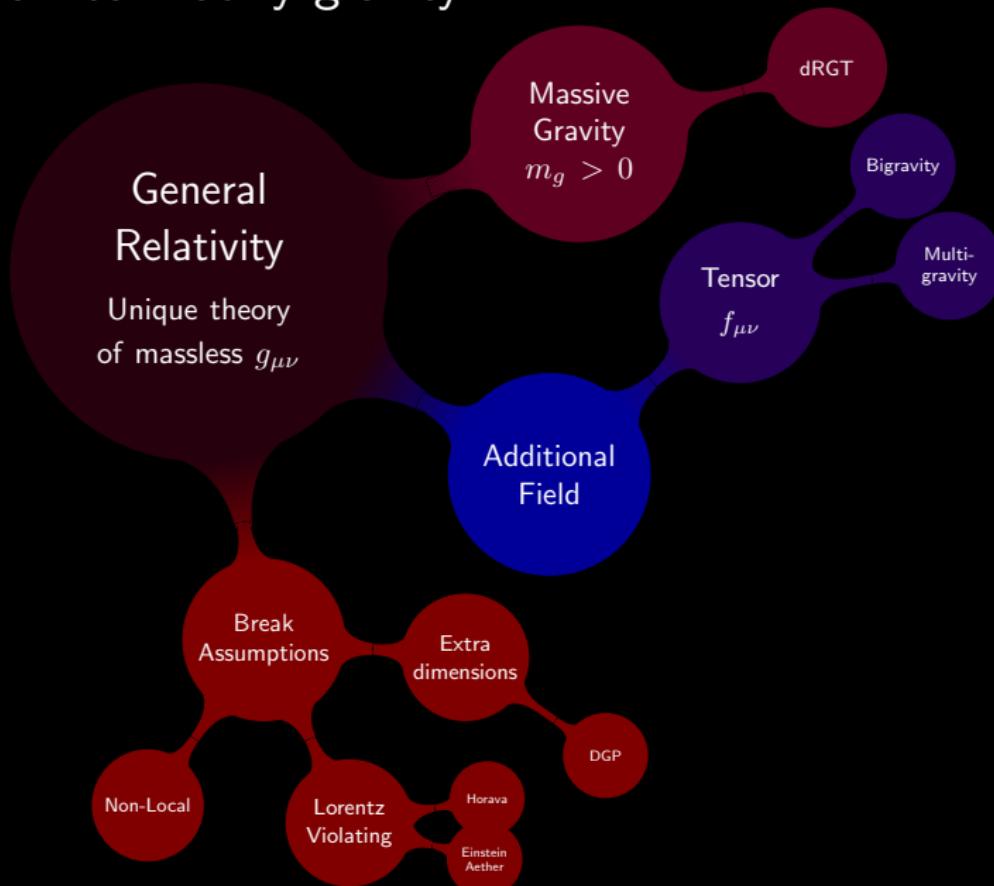
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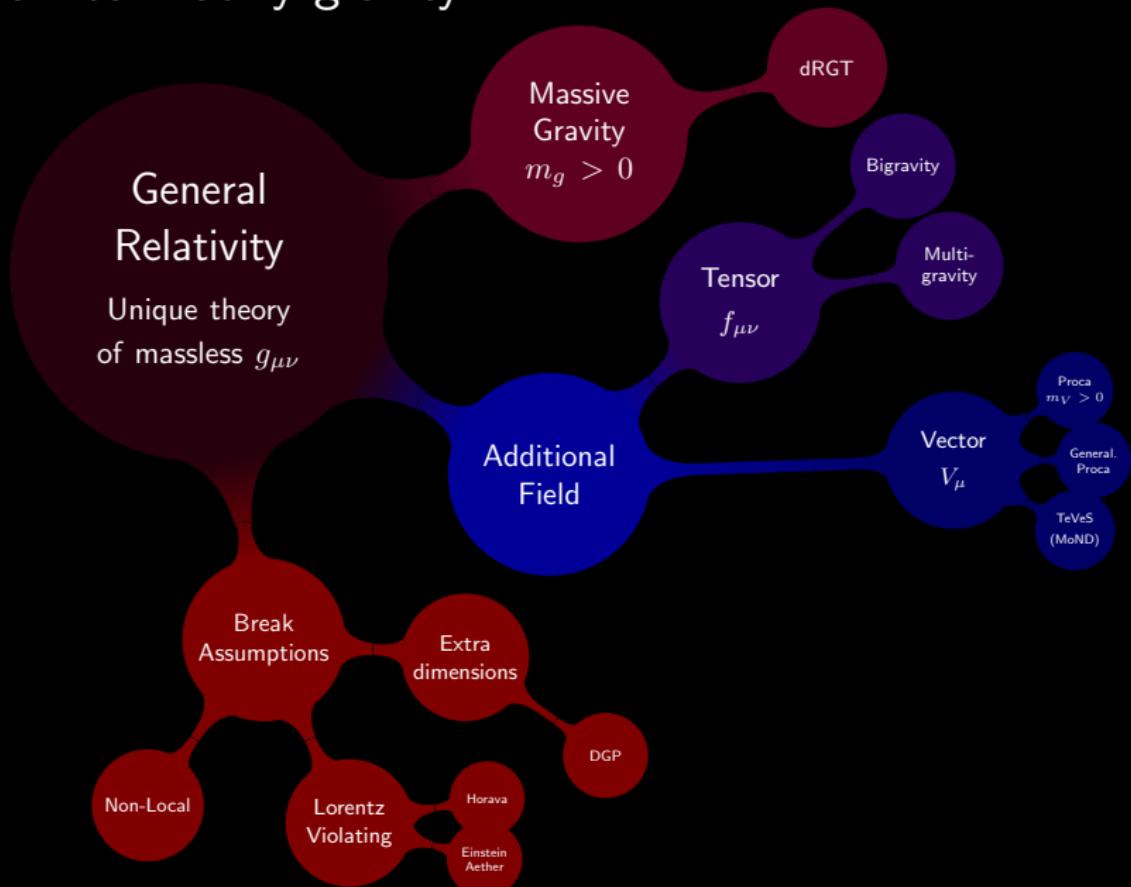
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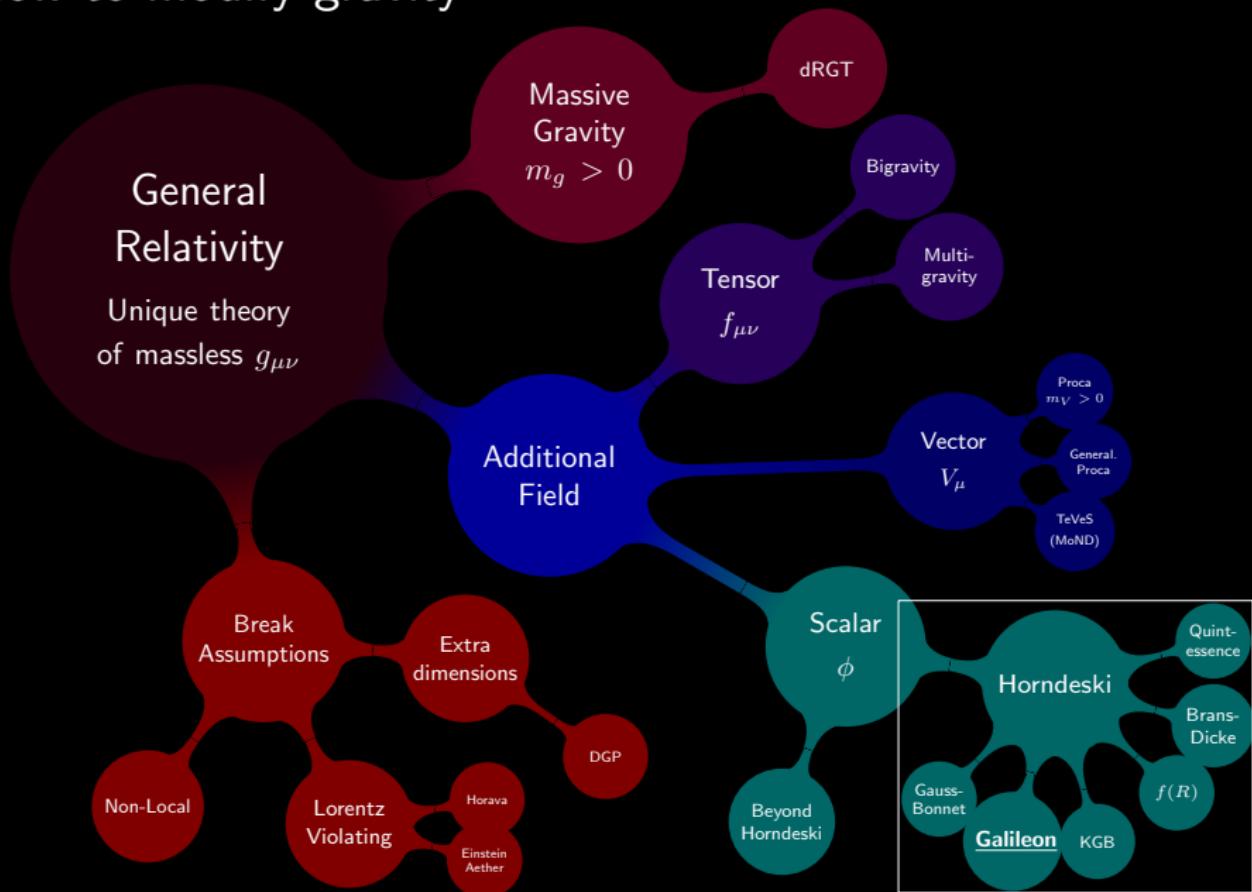
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# How to modify gravity



# Quintessence

(Wetterich '88, Rathra and Peebles '88)

$$S = \int d^4x \sqrt{-g} \left\{ \underbrace{\frac{1}{16\pi G} R}_{\text{metric}} [\underbrace{g_{\mu\nu}}_{\text{metric}}] + \underbrace{\frac{1}{2} (\partial\phi)^2}_{\text{kinetic}} - \underbrace{V(\phi)}_{\text{potential}} \right\}$$

Modifies cosmic expansion:

- $\rho_q = \frac{1}{2}\dot{\phi}^2 + V(\phi)$      $p_q = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

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Modifies cosmic expansion:

- $\rho_q = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad p_q = \frac{1}{2}\dot{\phi}^2 - V(\phi)$

- Acceleration equation:

$$\frac{\ddot{a}}{a} \propto -(\rho_m + 3p_m) - 2\dot{\phi}^2 + 2V(\phi)$$

- Slow roll  $\dot{\phi} \ll \sqrt{V} \Rightarrow \text{effective } \Lambda$

$$w \equiv \rho_q/p_q \approx -1 + \dot{\phi}^2/V \quad \Rightarrow \quad \underline{w > -1}$$

$\Rightarrow$  Probed by SNe,  $H_0$ , BAO, CMB scales...

# Brans-Dicke theory

(Jordan '59, Brans & Dicke '61)

$$S = \int d^4x \sqrt{-g} \left\{ \underbrace{\frac{\phi}{16\pi G}}_{\text{effective } G} R[g_{\mu\nu}] + \underbrace{\frac{\omega_{\text{BD}}}{\phi}}_{\text{coefficient}} \underbrace{\frac{1}{2}(\partial\phi)^2}_{\text{kinetic}} \right\}$$

Modifies expansion and growth:  $g_{\mu\nu} \rightarrow -(1 + \Psi)dt^2 + (1 - \Phi)d\vec{x}^2$ ,  
 $\phi \rightarrow \phi_0 + \delta\phi$

- Scalar force:  $\vec{\partial}^2 \delta\phi = \frac{8\pi G}{2\omega_{\text{BD}} + 3} (\rho - 3p) + \dots$

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- Newtonian potential  $\rightarrow$  Probed by Clustering & RSD

$$\vec{\partial}^2 \Psi = \frac{8\pi G}{\phi_0} \rho + \vec{\partial}^2 \delta\phi + \dots$$

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$$\vec{\partial}^2 \Psi = \frac{8\pi G}{\phi_0} \rho + \vec{\partial}^2 \delta\phi + \dots$$

- Anisotropic stress  $\rightarrow$  Probed by Gravitational Lensing

$$\vec{\partial}^2 (\Psi - \Phi) = \vec{\partial}^2 \delta\phi + \frac{8\pi G}{\phi_0} \sigma + \dots$$

# Galileon gravity

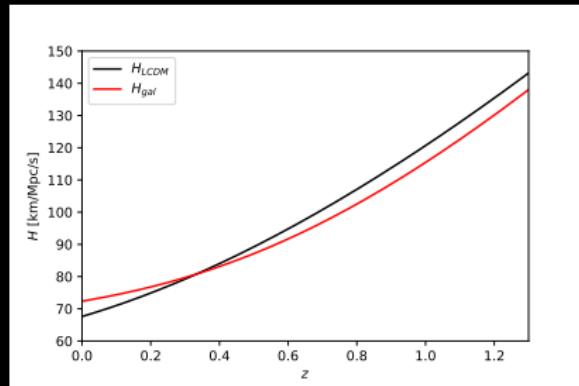
(Nicolis+ '08, de Rham Tolley '10...)

Limit of massive gravity, DGP/extra-dimensions:

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G} R[g_{\mu\nu}] - \underbrace{\frac{1}{2}(\partial\phi)^2}_{\text{kinetic}} + \underbrace{c_3(\partial\phi)^2 \square\phi}_{\text{deriv. int.}} + \dots \right\}$$

## Self-acceleration

- $\rho_{gal} = \frac{\Omega_{gal} H_0^2}{H(t)^2} \Rightarrow w < -1$
- $H^4 = H^2 \rho_m(t) + \Omega_{gal} H_0^4$   
accelerates with  $\Lambda = 0$



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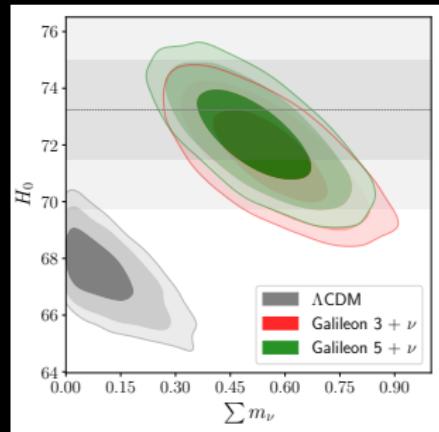
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- $H^4 = H^2 \rho_m(t) + \Omega_{gal} H_0^4$   
accelerates with  $\Lambda = 0$

- No  $\Lambda$ +GR limit  
 $\rightarrow$  very predictive



- Cosmologically viable (CMB+BAO) (Renk+ 1707.02263)
- Modified gravity effects hidden on small scales (screening)

# Scalar-Tensor gravity

- ★ Old-School:  $\frac{f(\phi)R}{16\pi G} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \supset \text{Quintessence/Inflation},$   
 $\supset \text{Brans-Dicke, } f(R) \quad (\text{Jordan '59, Brans & Dicke '61})$

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## ★ Horndeski's Theory (1974)

$g_{\mu\nu} + [\phi] + \text{Local} + 4\text{-D} + \text{Lorentz theory with } [2^{nd} \text{ order Eqs.}]$

$4 \times$  functions  $G_i(X, \phi)$  of  $\phi$ ,  $X \equiv -(\partial\phi)^2/2$

$$\mathcal{L}_H = G_2 - G_3 \nabla^2 \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

$\supset$  GR, quint/k-essence, Brans-Dicke,  $f(R)$ , chameleons...

kinetic gravity braiding, covariant Galileon, Gauss-Bonnet...

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- ★ Old-School:  $\frac{f(\phi)R}{16\pi G} - \frac{1}{2}(\partial\phi)^2 - V(\phi) \supset \text{Quintessence/Inflation},$   
 $\supset \text{Brans-Dicke, } f(R) \quad (\text{Jordan '59, Brans & Dicke '61})$

## ★ Horndeski's Theory (1974)

$g_{\mu\nu} + [\phi] + \text{Local} + 4\text{-D} + \text{Lorentz theory with } [2^{nd} \text{ order Eqs.}]$

4× functions  $G_i(X, \phi)$  of  $\phi$ ,  $X \equiv -(\partial\phi)^2/2$

$$\mathcal{L}_H = G_2 - G_3 \nabla^2 \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

$\supset \text{GR, quint/k-essence, Brans-Dicke, } f(R), \text{ chameleons...}$

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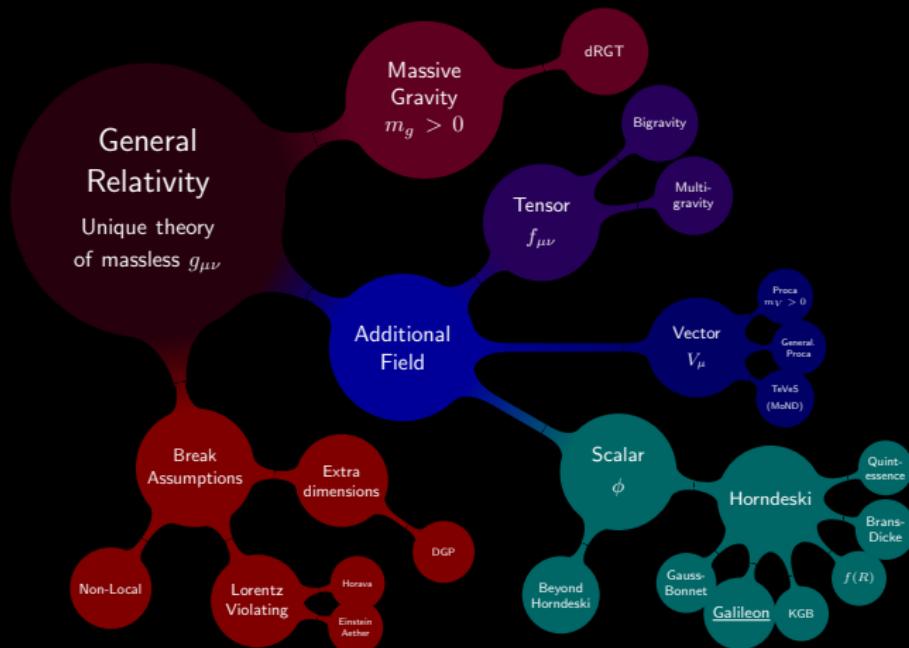
kinetic gravity braiding, covariant Galileon, Gauss-Bonnet...

- ★ Beyond Horndeski  $\rightarrow$  *discovered trying to simplify  $\mathcal{L}_H$ !*

(MZ & Garcia-Bellido '13, Gleyzes *et al.* '14, Langlois & Noui '15, ...)

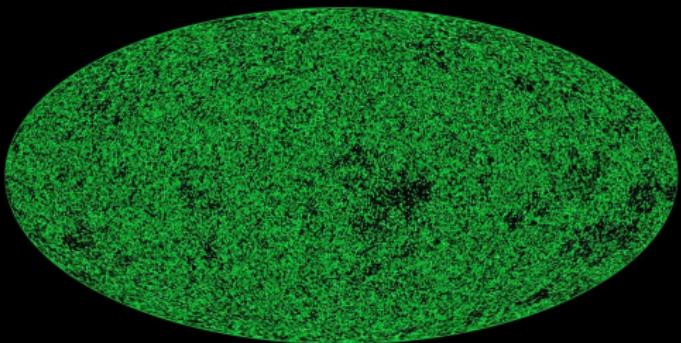
# How to test gravity

We have too many theories!



⇒ narrow the scope: focus on **cosmology**, solar system, GWs...

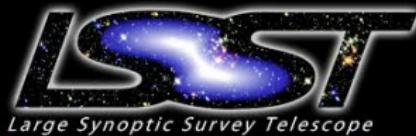
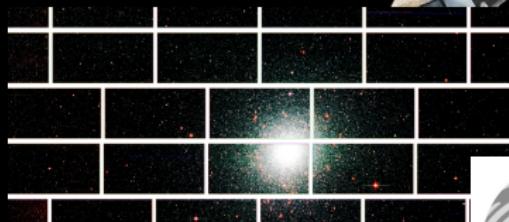
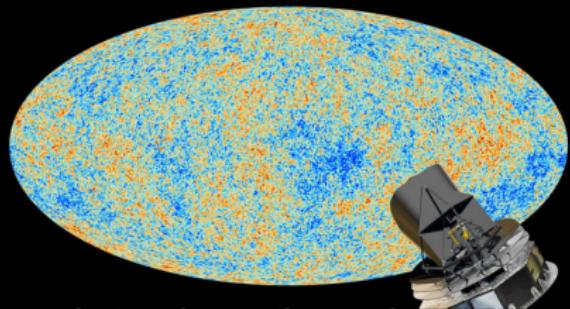
# Cosmological Tests



of Gravity and Dark Energy

PLANCK

# Precision Cosmology: new surveys planned



*Large Synoptic Survey Telescope*



**euclid**



Tests of Gravity & DE with cosmology & GWs

images from Planck, SDSS, Dark Energy Survey

Miguel Zumalacárregui (Berkeley)

# Test gravity: 3 approaches

## 1 - Model from Lagrangian

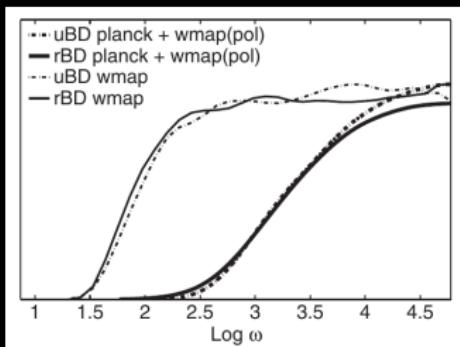
$$\underbrace{\sqrt{-g} \left\{ \frac{1}{16\pi G} R[g_{\mu\nu}] + \mathcal{L}_m \right\}}_{\text{Theory (Lagrangian)}} \rightarrow \underbrace{G_{\mu\nu} = 8\pi G T_{\mu\nu}}_{\text{Equations}} \rightarrow \underbrace{H(t), P(k, t)}_{\text{Solutions}}$$

- Specific, self-consistent
- Variable freedom: parameters + ICs  $\rightarrow$  several free functions
- Fully predictive: expansion history, N-body, GWs...

Example: Brans-Dicke

$$\mathcal{L} \propto \phi R - 2\Lambda - \frac{\omega}{\phi} (\partial\phi)^2$$

(Avilez & Skordis '14)



# Test gravity: 3 approaches

- Model from Lagrangian

## 2 - Parameterize the solutions

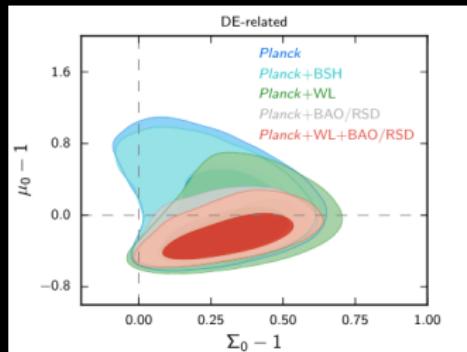
$$\nabla^2 \Psi = 4\pi G a^2 \mu(a, k) \rho \Delta, \quad \nabla^2(\Phi + \Psi) = 8\pi G a^2 \Sigma(a, k) \rho \Delta$$

- Fully general
- Vast functional freedom: 2 functions of 2 variables
- Only linear regime, no expansion history

Example: (Planck '15 DE paper)

$$\mu = 1 + \mu_0 \Omega_{de}(a)$$

$$\Sigma = 1 + \Sigma_0 \Omega_{de}(a)$$



# Test gravity: 3 approaches

- Model from Lagrangian
- Parameterize the solutions

## 3 - Effective theory approaches

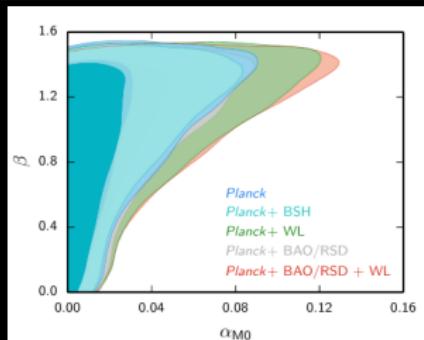
$$\mathcal{L} = \sum_i \alpha_i(t) \mathcal{O}_i$$

- Rather general: locality, covariance, d.o.f. # & type
- Large functional freedom  $\mathcal{O}(\text{few})$  functions, 1 variable
- Limited info from other regimes GWs (no expansion history)

Example: (Planck '15 DE paper)

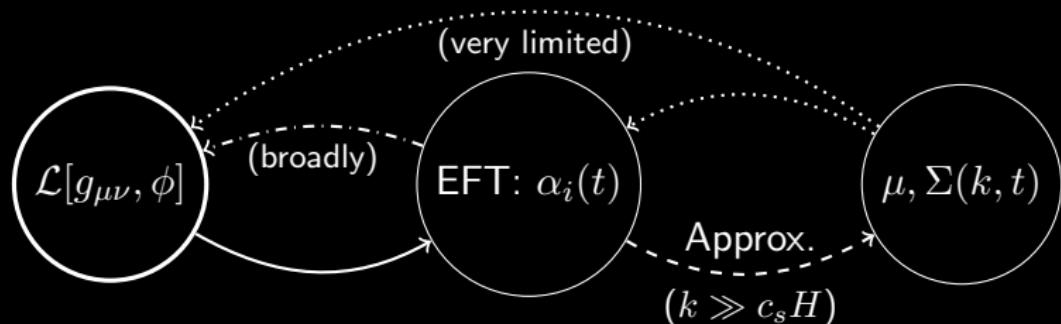
$$\alpha_M = \frac{a}{\Omega + 1} \frac{d\Omega}{da} = -\alpha_B$$

$$\Omega(a) = \exp \left[ \frac{\alpha_{M,0}}{\beta} a^\beta \right] - 1$$



# Connecting approaches

(Gubitosi+, Bloomfield+ '14)



## Remarks:

- (Predictions,  $\rightarrow$ ) Easy to go from concrete to generic
- (Data,  $\leftarrow$ ) Hard to infer theory from parameterization
- Specific is always more info (GWs, non-linear)
- EFT is a good middle ground

# Horndeski in EFT language

(Bellini & Sawicki '14)

$$\underbrace{\ddot{h}_{ij} + 3H(1+\alpha_M)\dot{h}_{ij}}_{\delta(\sqrt{-g}M_*^2\dot{h}_{ij}^2)} + \underbrace{(1+\alpha_T)k^2 h_{ij}}_{c_T^2, \text{ GW}} = 0 \quad (\text{tensors})$$

$$\underbrace{\alpha_K}_{\text{diagonal}} \delta \ddot{\phi} + 3H \underbrace{\alpha_B}_{\text{mixing}} \ddot{\Phi} + \underbrace{(\dots)}_{\alpha_K, \alpha_B, \alpha_M, \alpha_T} = 0 \quad (\text{scalar field})$$

Theory-specific predictions:

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

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Kineticity:  $\alpha_K$

Standard kinetic term  $\rightarrow c_S^2$

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Variation rate of effective  $M_p$

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Variation rate of effective  $M_p$

Braiding:  $\alpha_B$

Kinetic Mixing of  $g_{\mu\nu}$  &  $\phi$

Tensor speed excess:  $\alpha_T$

GW at  $c_g^2 = 1 + \alpha_T$

# From Data to Theory

$$G_2 - G_3 \square \phi + G_4 R + G_{4,X} [\nabla \nabla \phi]^2 + G_5 G_{\mu\nu} \phi^{\;\mu\nu} - \frac{G_{5,X}}{6} [\nabla \nabla \phi]^3$$

Kineticity:  $\alpha_K$

kinetic term  $\rightarrow c_S^2$

Braiding:  $\alpha_B$

Mixing of  $g_{\mu\nu}$  &  $\phi$

scalar  
&  
tensor

$M_p$  running:  $\alpha_M$

Variation of eff.  $M_p$

Tensor speed excess:  $\alpha_T$

GW speed  $c_T^2 = 1 + \alpha_T$

tensor  
only

Theory-specific relations, e.g.

- Quintessence:  $\alpha_K \propto (1+w)\Omega_{\text{DE}}$
- Brans-Dicke:  $\alpha_K, \alpha_B = -\alpha_M$
- Galileon  $G_3 \rightarrow \alpha_K, \alpha_B \neq \alpha_M$        $G_4, G_5 \rightarrow \alpha_M, \alpha_T$

Measure  $\alpha$ 's  $\leftrightarrow$  Properties of gravity & underlying theory

# EFT of Horndeski in practice

$$\left. \begin{array}{c} G_2, G_3, G_4, G_5 \\ \phi(t_0), \dot{\phi}(t_0) \end{array} \right\} \rightarrow \left. \begin{array}{c} \text{Kineticity } \alpha_K \\ \text{Braiding } \alpha_B \\ M_p \text{ running } \alpha_M \\ \text{Tensor speed } \alpha_T \end{array} \right\} \rightarrow \left. \begin{array}{c} D_A(z) \\ C_\ell \\ P(k) \\ \dots \end{array} \right.$$

a) Full theory + IC

b)      or      Parameterize  $w(z), \alpha_i(z)$

Full theory has more info

- Background  $\longrightarrow$  often very constraining
- Non-linear effects
- Other regimes: GWs, strong gravity, Solar System, QM, Lab...

# EFT tests

(Alonso, Bellini, Ferreira, MZ '16)

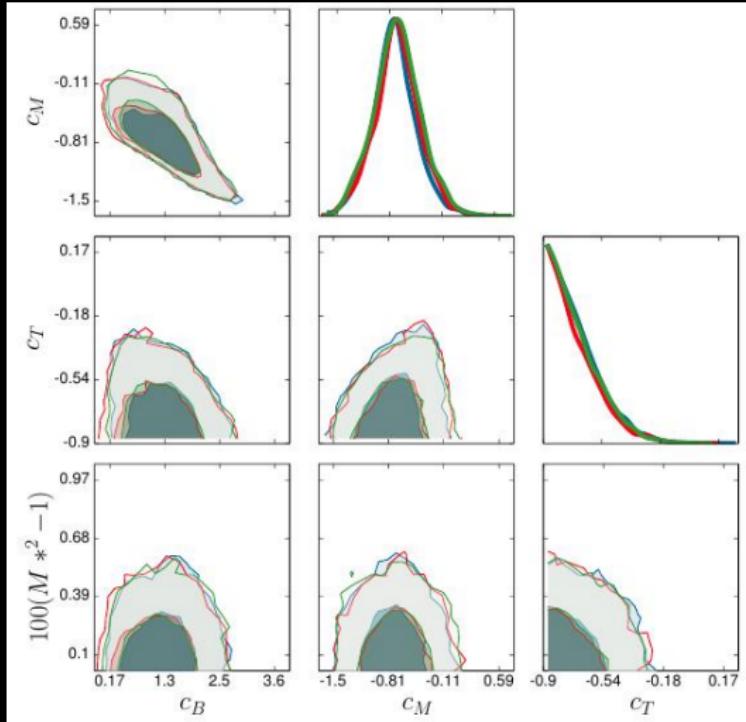
Horndeski parameterization

- $\alpha_i = c_i \cdot \Omega_{de}(z)$

- Current: (Bellini+ '15)

$$\Delta c_i \sim 1$$

CMB+BAO+RSD+PK



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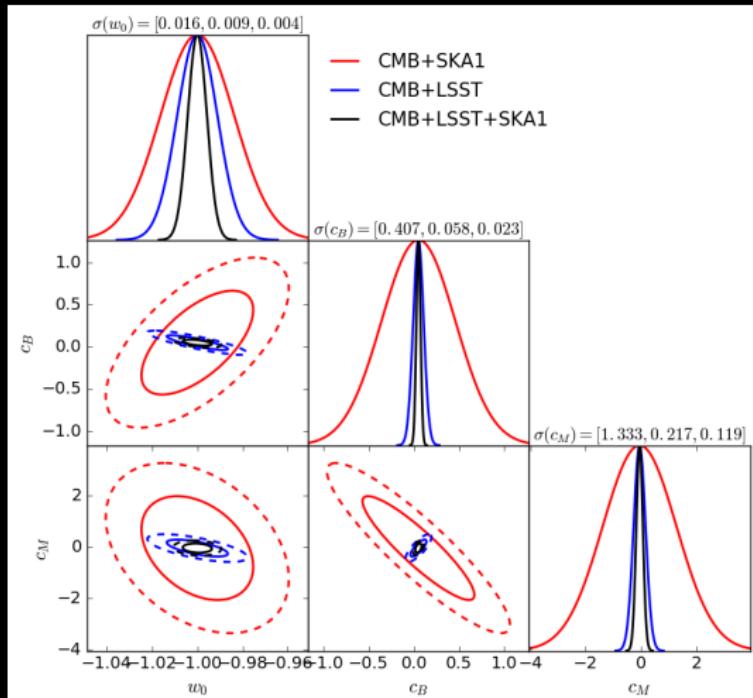
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- Future:

$$\Delta c_i \sim 0.1 - 0.01$$

- CMB S4 + SKA
- + LSST
- + DESI-like



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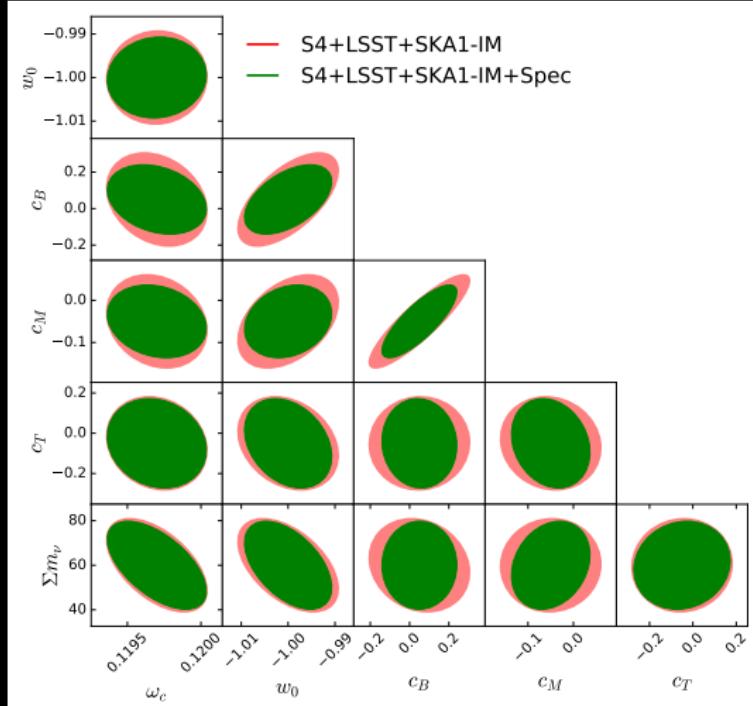
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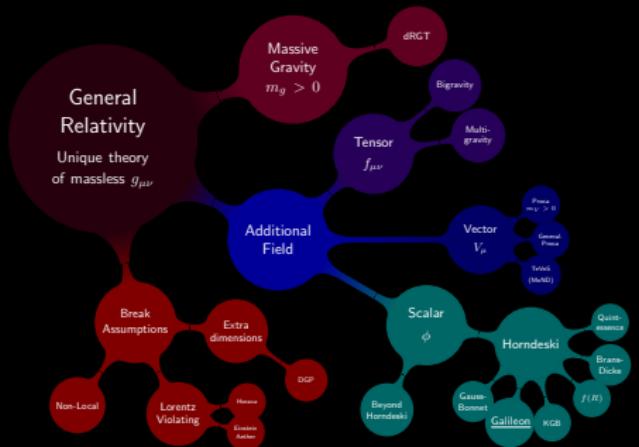
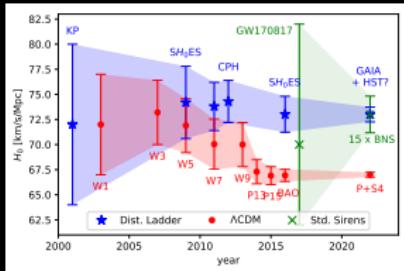
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# Conclusions

- Gravity is important:  $\exists$  good reasons for beyond GR
- Tensions in  $\Lambda$ CDM: new physics?
- Many theories available
  - general frameworks
  - flexible tools
- Most theories very predictive!
- Test theories in every regime: cosmology, local gravity, GWs...



# Backup Slides

$$\begin{array}{cccccccccc}
\Omega & \sqrt{-g} & \mathcal{L}_H & \alpha_B & \Psi & P & \Phi & \Omega & \sqrt{-g} & \mathcal{L}_H & \alpha_B & \Psi & \rho & \delta & R_{\mu\nu} & \delta & \phi_{\mu\nu} & h_+ \\
G_2 & \Phi & \alpha_M & \delta & P & \alpha_K & G_3 & G_2 & \Phi & \alpha_M & \delta & P & \alpha_K & \mathcal{H} & \delta & \phi_{\mu\nu} & \alpha_M & \Psi & h_+ \\
\delta & R_{\mu\nu} & c_T^2 & \phi_{\mu\nu} & h_+ & c_T^2 & X & G_2 & R_{\mu\nu} & c_T^2 & \phi_{\mu\nu} & h_+ & \mathcal{H} & \phi_{\mu\nu} & \alpha_M & \Gamma_{\mu\nu}^{\rho} & R & \alpha_H \\
\mathcal{H} & \delta & \Psi & h_+ & \alpha_T & c_T^2 & X & h_+ & \delta & R_{\mu\nu} & c_T^2 & \phi_{\mu\nu} & h_+ & \mathcal{H} & \phi_{\mu\nu} & \alpha_M & \Gamma_{\mu\nu}^{\rho} & R & \mathcal{E} \\
\phi_{\mu\nu} & \alpha_M & \mathcal{H} & \delta & \phi_{\mu\nu} & \square \phi & \alpha_T & \alpha_M & \mathcal{H} & \alpha_K & \delta & \mathcal{P} & X & G_1 & L_H & \alpha_X & G_1 \\
\Phi & \Gamma_{\mu\nu}^{\rho} & \Pi & \alpha_K & \mathcal{E} & \Pi & \alpha_T & \Phi & \Gamma_{\mu\nu}^{\rho} & \alpha_M & \mathcal{H} & \alpha_K & \delta & \mathcal{P} & X & G_1 & L_H & \alpha_X & G_1 \\
X & G_1 & \mathcal{L}_H & \alpha_K & \mathcal{E} & \Pi & \alpha_T & \Phi & \Gamma_{\mu\nu}^{\rho} & \Pi & \mathcal{H} & \alpha_K & \delta & \mathcal{P} & X & G_1 & L_H & \alpha_X & G_1 \\
w & k^2 & X & \delta & \phi_{\mu\nu} & G_1 & \sqrt{-g} & w & k^2 & X & \mathcal{H} & G_2 & \alpha_K & \alpha_K & \square \phi & G_2 & \phi_{\mu\nu} & G_2 \\
V_X & G_2 & \square \phi & \mathcal{H} & R & \alpha_M & \Pi & V_X & G_2 & \square \phi & G_2 & \alpha_K & \alpha_K & \square \phi & G_2 & G_1 & \Phi & X \\
G_2 & c_T^2 & \mathcal{L}_H & G_5 & G_6 & \Psi & G_2 & c_T^2 & \mathcal{L}_H & \Phi & \phi_{\mu\nu} & c_T^2 & R & R_{\mu\nu} & \alpha_B & G_5 & c_T^2 & M_*^2 \\
\alpha_K & \square \phi & \phi_{\mu\nu} & \Phi & V_X & G_1 & \alpha_K & \square \phi & G_1 & \alpha_B & \alpha_B & R_{\mu\nu} & \alpha_B & G_5 & G_5 & h_+ \\
G_2 & G_2 & \phi_{\mu\nu} & \theta & G_1 & \alpha_K & \square \phi & G_1 & \phi_{\mu\nu} & G_5 & G_5 & R_{\mu\nu} & \alpha_B & G_5 & G_5 & h_+ \\
\delta & \Phi & c_T^2 & \theta & X & \alpha_M & G_1 & \alpha_B & \theta & X & \alpha_B & \theta \\
\alpha_B & G_5 & \phi_{\mu\nu} & \alpha_B & M_*^2 & & & & & & & & & & & & & & & M_*^2 \\
& h_+ & & & c_T^2 & & & & & & & & & & & & & & & & & M_*^2
\end{array}$$

hi\_class

$$\begin{array}{cccccccccc}
h_+ & \Psi & & & & k^2 & & & & R_{\mu\nu} \\
\mathcal{H} & \phi_{\mu\nu} & & & & X & & & & h_+ \\
G_5 & \alpha_B & \square \phi & & & \sqrt{-g} & \alpha_B & \delta & \alpha_H & \alpha_M \\
\mathcal{L}_H & \alpha_B & \Phi & G_1 & & X & \Pi & G_1 & L_H & \alpha_T & G_4 & \alpha_M & \sqrt{-g} \\
\mathcal{H} & \alpha_M & V_X & \mathcal{L}_H & \sqrt{-g} & \delta & & & & h_+ & \Phi & \alpha_K & c_T^2 & \alpha_B \\
\sqrt{-g} & \alpha_B & \Psi & \rho & & & & & & & & & & & & & & & & & \\
\Phi & \delta & P & \alpha_K & G_5 & \Phi & R & c_T^2 & G_4 & \mathcal{L}_H & \alpha_K & M_*^2 & \mathcal{L}_H & \alpha_T & G_4 & \Phi & \Gamma_\rho^0 \\
R_{\mu\nu} & c_T^2 & \phi_{\mu\nu} & h_+ & \mathcal{H} & R_{\mu\nu} & \phi_{\mu\nu} & X & \alpha_M & \theta & G_2 & \phi_{\mu\nu} & \alpha_B & R_{\mu\nu} & \Phi & \Gamma_\rho^0 \\
\bar{\phi} & \Psi & h_+ & \alpha_T & \Omega & c_T^2 & \mathcal{L}_H & \Psi & k^2 & \alpha_T & \square \phi & \mathcal{P} & \delta & \Pi & \Psi & G_5 & h_+ & \Phi_3 \\
\alpha_M & \mathcal{H} & \alpha_K & c_T^2 & \mathcal{P} & \theta & \Gamma_{\mu\nu}^{\rho} & \alpha_K & \Pi & c_T^2 & \Psi & R_{\mu\nu} & c_T^2 & \square \phi & k^2 & \mathcal{E} & \Psi \\
\Gamma_{\mu\nu}^{\rho} & \Pi & \delta & \mathcal{E} & \Gamma_{\mu\nu}^{\rho} & \Phi & X & \mathcal{P} & \mathcal{L}_H & \Phi & X & G_3 & \sqrt{-g} & \Phi & G_3 & R & X & \theta \\
G_4 & \mathcal{L}_H & \delta & G_4 & X & h_+ & \delta & \square \phi & \Pi & R & \mathcal{E} & \alpha_B & \mathcal{L}_H & \phi_{\mu\nu} & \delta & \Psi & \sqrt{-g} & \mathcal{L}_I \\
k^2 & X & \mathcal{H} & \phi_{\mu\nu} & \alpha_B & \alpha_T & G_1 & \mathcal{L}_H & X & \Omega & h_+ & M_*^2 & h_+ & \Omega & G_2 & \mathcal{L}_H & c_T^2 & G
\end{array}$$

[www.hiclass-code.net](http://www.hiclass-code.net)

# Effective theory of Dark Energy (Gubitosi+, Bloomfield+ '13)

- Too many gravity theories  $\Rightarrow$  systematic approach
- Background:  $w(z) \rightarrow$  complete description

$$p_{DE} = w(z)\rho_{DE}, \quad \dot{\rho}_{DE} + 3H(1+w)\rho_{DE} = 0$$

$\rightarrow w(z)$  with data (parameterization, binning, principal components...)

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- EFT-DE  $\rightarrow$  Most general theory of gravity with:

★ Only background + linear perturbations

★ Tensor + scalar field ...

★ FRW Symmetries: homogeneity + isotropy

★ Theory symmetries: coordinate transformations

$\Rightarrow$  Finite set of  $\alpha_i(z)$  functions  $\leftrightarrow$  describes any theory

(Review: Gleyzes, Langlois, Vernizzi '14)